EFFECTIVE TEACHING OF MATHEMATICS

Conceptual understanding…

- “the comprehension and connection of concepts, operations, and relations” (National Council of Teachers of Mathematics [NCTM], 2014)

- “a deep understanding of the concept and its components” (Rittle-Johnson & Schneider, 2014)

- establishes the foundation, and is necessary, for developing procedural knowledge

- supports retention and transfer of knowledge
HOW TO TEACH MATHEMATICS EFFECTIVELY

- From direct teaching to reform-oriented teaching

- Problem-solving
  - Students construct knowledge
  - Students struggle is part of the process
STUDENTS’ OPPORTUNITY TO STRUGGLE

- Struggle with important mathematics
  - Not frustration or despair

- “The effort students expend to uncover new relationships within and gaining insights about the structure of mathematics” (Hiebert & Grouws, 2007)
PRODUCTIVE STRUGGLE

- Struggle itself
  - doesn’t guarantee conceptual understanding
  - isn’t always productive (e.g. Kapur, 2010; Stein, Grover, & Henningsen, 1996)

- Question: What makes struggle productive?
TWO INSTRUCTIONAL DESIGNS TO PROMOTE PRODUCTIVE STRUGGLE

Mathematics Education: 
**Launch-Explore-Summarize**

- **Launch phase**
- **Explore phase**
- **Summarize phase**

(Cognitive Science: Problem Solving First)

(e.g. Lappan et al., 2010, 2014; Smith & Stein, 2011)

(e.g. Kapur, 2014, 2016; Schwartz & Martin, 2004)
TWO INSTRUCTIONAL DESIGNS TO PROMOTE PRODUCTIVE STRUGGLE

Mathematics Education: Launch-Explore-Summarize

Launch phase

Explore phase

Summarize phase

(e.g. Lappan et al., 2010, 2014; Smith & Stein, 2011)

Cognitive Science: Problem Solving First

Problem-solving phase

Explicit instruction phase

(e.g. Kapur, 2014, 2016; Schwartz & Martin, 2004)
LAUNCH-EXPLORE-SUMMARIZE

- **Launch phase**
  - Problems are carefully selected and set up to invite and surface student thinking

- **Explore phase**
  - Students struggle to solve the problem
  - Teacher circulates and assists (scaffolding)

- **Summarize phase**
  - Teacher leads a whole-class discussion in which students actively engage
PROBLEM-SOLVING FIRST

Problem Solving phase
- Students struggle to solve the problem and they usually fail
- Prepares students for future learning

Explicit Instruction phase
- Students are “told” the canonical solution or the concept
COMMON FEATURES ACROSS TWO DESIGNS
COMMON FEATURES OF TEACHING THAT PROMOTES PRODUCTIVE STRUGGLE

Use challenging mathematical problems

- Launch-Explore-Summarize:

  Mathematical Task Analysis Guide*
  by Stein, Smith, Henningsen, and Silver (2009)

Cognitive challenge of instructional tasks

- Low-level (i.e. memorization and procedures without connections)
  - routine problems
  - students simply execute standard algorithms

- High-level (i.e. procedures with connections and doing mathematics)
  - Multiple solution strategies
  - Multiple representations
  - Students make connections
“Compute the perimeter for the first four trains, determine the perimeter for the tenth train without constructing it, and write a description that could be used to compute the perimeter of any train in the pattern. Use the edge length of any pattern block as your unit of measure.”

An example of high-level task (Boston, 2012)
COMMON FEATURES OF TEACHING THAT PROMOTES PRODUCTIVE STRUGGLE

Use challenging mathematical problems

- Problem-solving first:
  - Ill-structured problems (e.g. Kapur, 2012)
  - Contrasting cases (e.g. Schwartz & Martin, 2004)
CHALLENGING PROBLEMS USED IN PROBLEM-SOLVING FIRST DESIGN

- Generate multiple representations and solution strategies
- Make and justify assumptions
- Activate prior knowledge
- Prepare for future learning
INTERACTIVE REPRESENTATIONAL SYSTEMS MODEL

Lesh (1979)
COMMON FEATURES OF TEACHING THAT PROMOTES PRODUCTIVE STRUGGLE

- Order of the phases is important

- Students start the lesson by working on the problems without immediate assistance from the teacher
WHY IS ORDER IMPORTANT?

Mathematics education:

- Students explore the problem and try out different ideas to come up with their own solutions
- Students have the chance to develop arguments and justifications for their thinking

Cognitive science:

- Problem-solving creates awareness of knowledge gap
- Direct instruction fills the knowledge gap
WHY IS ORDER IMPORTANT?

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COMMON FEATURES OF TEACHING THAT PROMOTES PRODUCTIVE STRUGGLE

- Teachers build instruction on students’ solution strategies
  - Connections between different students’ solutions
  - Connections between students’ solution strategies to the canonical solution
Problem-solving first approach prompt students to...

- activate their prior knowledge (Mechanism 1)
- become aware of their specific knowledge gaps (Mechanism 2)
- and encode knowledge in relation to deep features (Mechanism 3)

This figure shows the interaction between these mechanisms (Loibl & Rummel, 2017)
5 PRACTICES (MATH & SCIENCE)

- Anticipating students’ solutions to a mathematics task
- Monitoring students’ in-class, “real-time” work on the task
- Selecting approaches and students to share them
- Sequencing students’ presentations purposefully
- Connecting students’ approaches and the underlying mathematics

Smith & Stein (2018)
WHAT MAKES STRUGGLE PRODUCTIVE?

- Teachers select problems for which students don’t have a known strategy
- Students start the lesson by working on the problems without immediate assistance
- Teachers build instruction on students’ solution strategies
THANK YOU!

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